Adaptive Ground Modelling in geotechnical engineering

M. Schönhardt & K. J. Witt
Professorship of foundation engineering, Bauhaus-University Weimar, Germany

ABSTRACT: Geotechnical design depends on parameters got from field investigations. Therefore the development of subsoil models for assessing the behaviour of the subsoil has a strong influence on the reliability of geotechnical structures. The contribution deals with an adaptive improvement of soil models using geostatistical methods, demonstrated for a data set of a simplified practical example. Starting with subjective interpolations based on two data sets from a preliminary a detail investigation, the influence of additional integrated geotechnical model assumption is shown. In a next step further regionally information are integrated into the model. This given soil information are necessary to calibrate the results on any location in the field using the smoothing splines method. For a virtual machine footing the influence of the generated model precision on the dimension of an assumed square size isolated machine foundation is analysed. The uncertainties and spread of all characteristic quantities in the limit state function, an equation for determine the skew position of the footing, is taken into account. Using the Monte Carlo Simulation for uncorrelated values, the reliability of the isolated foundation is determined. Finally, the different predictions of the adaptive modelling are compared with the real soil conditions at the site.

1 INTRODUCTION

One of the most essential tasks of geotechnical engineering is to develop a geological model of the subsoil. With different techniques we make a limited number of observations and measurements and we draw scientifically defensible conclusions from them. Based on different field monitoring data we have to build up a geological model of the site conditions, describing the layers, kind of soil, mechanical properties as well as the spatial variability of those parameters. The reliability of such a model is fundamental for all decisions during design procedure. Thus, the interpretation of field data requires insight into the geology, its properties, details and anomalies but also experience with foundation engineering and construction practice. In the stage of site investigation we usually deal with two typical problems (i) very few observations and measurements due to limited budged and (ii) only rough information about the structure of the building, the quantity and location of loads. Therefore we have to build up the geological model in a more or less adaptive manner, using successive steps of investigation as well as geostatistical methods for controlling, supporting and improving the interpretation. These steps should be demonstrated with a simplified practical example.

For planning a new large industrial plant, the underground was investigated at a stage with less information about the structure, the number and size of the machines, their loads and their foundations. Figure 1 shows a map of the machine hall with some possible areas for the machine placement. From a preliminary course of investigation, using hand driven auger drillings, dynamic probing and some core drillings, we got general data about the subsoil of the overall area. This step delivers level 1 information. In a detailed second level focussed on the machine hall we carried out some more and deeper core drillings and a couple of static cone penetration tests in a systematic screen. All points of investigation are marked in figure 1.

Evaluating and assessing the test results the subsoil surround the machine hall can be subdivided in 3 relatively homogeneous layers with a more or less random stratum: A thin upper silty clay with a soft consistency, a second layer out of a stiff clay reaching down to 8 –14 m and under that until a depth of 70 m a sandy clay with a very stiff and hard consistency.

The data of the different steps of exploration are used for an adaptive modeling of homogeneous profiles with help of geostatistical methods. Thus we can demonstrate the influence of limited investigation to the reliability of the prediction. At least we can use this certain levels of prediction to design a virtual isolated footing in the centre of the machine hall in a probabilistic manner. To compare the prediction under certain levels of information with the reality, the virtual footing was placed at a point of investigation without using this real data in the process of prediction.

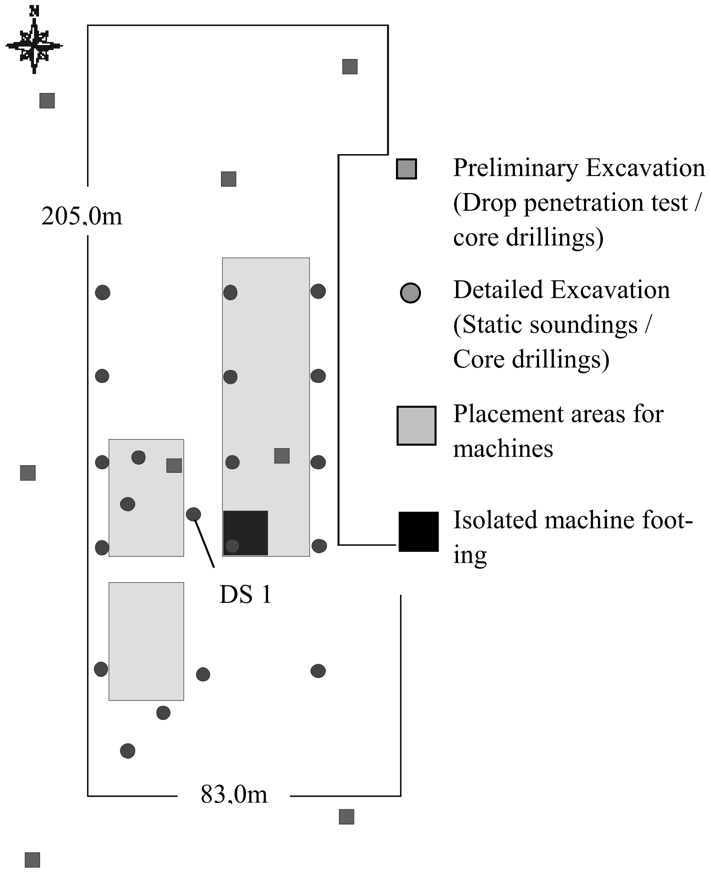


Figure 1. Arrangement of preliminary and detailed excavation/ Placement of the foundations

2 HOMOGENEOUS PROFILS

Starting with a subjective estimation of a general homogeneous profile on the base of the preliminary and the detailed investigation the homogeneous profile is specified in the single adaptive steps supported by geostatistical methods under consideration of scattering values. The scattering values and its uncertainties are integrated trough a wide range of repetition in the numerical computation. Every parameter set is different from each other. Every quantity is determined by there characteristic value, the standard deviation and the statistical distribution (SCHÖNHARDT 2002).

At first the homogenous profile was estimated subjective using the results of the preliminary investigation. After that the data set has been extended trough the data of the cone penetration. This leads to a new homogeneous profile. The reliability of both data sets shall be equal, so that no weighting factors a necessary for bringing together the different field investigations.

The next adaptive step will take the results of the level 1 investigation. By using geostatistical methods on the base of variographie and universal kriging the homogeneous profile for the foundation was determined more precisely.

In the next step this soil model was extended trough the data set of the detailed investigation.

In a further step of the adaptive modelling a consistency proving method was integrated. Here additional regional information in form of a given homogeneous stratification is necessary.

Finally the different created homogeneous predictions are compared with the real soil profile on the foundation place. This information is derived by a separately static sounding.

3 PROCEDURE OF MODELLING

The results of the preliminary investigation are used for a first soil model. The subjective arithmetically estimation of the thickness for each homogeneous layer is mostly too safe. The single estimations of the layers thickness are built into a 3d-soil model. The strata of the homogeneous surfaces are presented in table 1.

Table 1. Strata of the layers, using the preliminary investigation [m]

	stratum	stand. deviation
Layer 1	129,94	1,648
Layer 2	127,83	1,361
Layer 3	119,76	2,036

Specifically characteristic properties will be only conditional considerate. In the neighbourhood of the foundation we have only a low density of information. The second level of investigation improved this weak point.

From the new greater data set we derived in a second estimation a new set of the absolute heights of the homogeneous layers (table 2).

Table 2. Strata of the layers, using the preliminary and the detail investigation [m]

	stratum	stand. deviation
Layer 1	130,22	1,274
Layer 2	127,94	1,329
Layer 3	117,89	3,193

The evaluation of the results presented in table 1 and table 2 shows that the surfaces form layer 1 and layer 2 are similar. The stratum of the layer 3 is different. The difference is expressed in the standard deviation of the arithmetical determination. Both estimations contain uncertainties, because internal trend and the anisotropic behaviour within the different data are not considered.

A third estimation using geostatistical method takes the anisotropy and trends into account, CHRISTAKOS 1992. The result is an improved 3d-soil model. Internal computations are necessary to distinguish between local drift and global trend. The drift function is indicated by a linear combination, which extended the normal universal Kriging matrix.

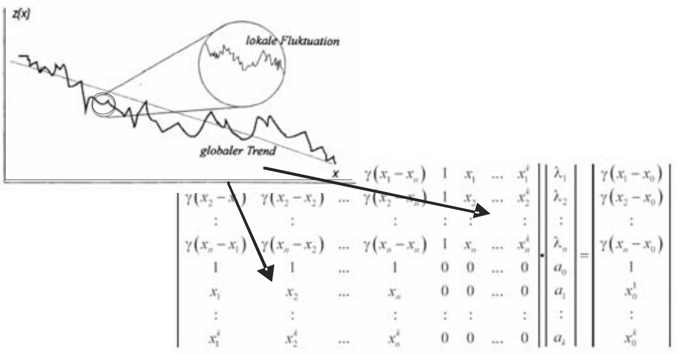
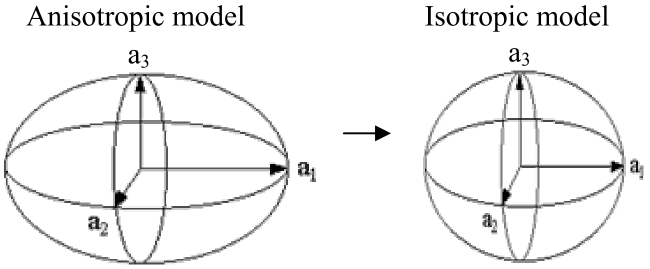


Figure 2. Drift function implemented in Kriging matrix SCHAFMEISTER 1999

The analysis of the data shows that there is a global trend and different drifts. The trend can be imposed by fluctuated values. In addition there is an anisotropy direction in the area. Every research area has an own anisotropy ratio a_1:a_2:a_3. In opposite of the trend is expected value is equally, but the spatial correlation is different. The spatial range in the variogramm computation differs. The anisotropic soil can be seen as an ellipsoid soil model, figure 3.


Anisotropic model

Isotropic model

Figure 3. Anisotropic and isotropic soil model simplification

The prerequisite for geostatistical computations is the isotropy, CLEMENS 1995. The main isotropy direction here is determined to 350° NNW. Therefore the actual soil properties must be transformed from the ellipsoid to the unit sphere.

After during that on the base of the whole date population from all levels using the self developed software application GEOSTAT a fourth and fifth homogeneous soil profile is generated. The homogeneous profile for the foundation place calculated in this manner, is given in table 3 and table 4 columns 1 and 2.

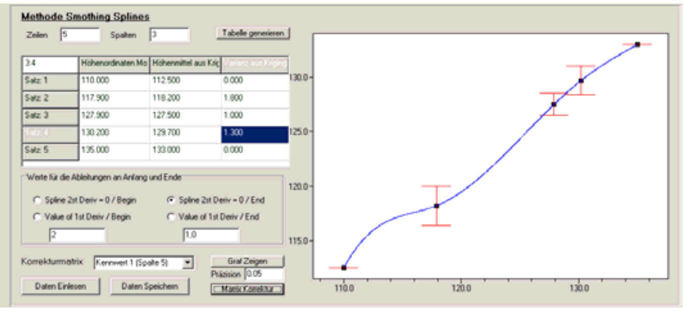
Table 3. Strata of the layers, using the preliminary investigation combined with geostatistical methods [m]

	stratum	standard deviation	stratum [consistent]	standard deviation [consistent]
	1	2	3	4
Layer 1	129,60	1,412	129,16	1,507
Layer 2	127,44	1,100	126,99	1,411
Layer 3	119,38	1,460	118,22	1,220

Table 4. Strata of the layers, using the preliminary and detail investigation combined with geostatistical methods [m]

	stratum	standard deviation	stratum [consistent]	standard deviation [consistent]
	1	2	3	4
Layer 1	129,73	1,251	129,21	1,148
Layer 2	127,48	1,040	127,06	0,929
Layer 3	118,22	1,810	118,18	0,950

The sum of thickness of the modelled layers differs. To guarantee the consistency is it necessary to involve further information. An additional given homogeneous profile will be integrated in the soil model using the method of smoothing splines. The thickness of the single layers will be corrected before they are put together in the model, figure 4.


Methode Smothing Splines

Zeilen 5 Spalten 3

3.4	Höhenordinaten Mo	Höhenmittel aus Krig	Varianz aus Kriging
Satz 1	110.000	112.500	0.000
Satz 2	117.900	118.200	1.800
Satz 3	127.900	127.500	1.000
Satz 4	130.200	129.700	1.300
Satz 5	135.000	133.000	0.000

Werte für die Ableitungen an Anfang und Ende

○ Spline 2st Deriv = 0 / Begin ● Spline 2st Deriv = 0 / End

○ Value of 1st Deriv / Begin ○ Value of 1st Deriv / End

2 1,0

Korrekturmatrix Kennwert 1 (Spalte 5)

Präzision 0.05

Figure 4. Exemplary use of the smoothing splines method as matrix adaptation scheme in GEOSTAT

The uncertainties and spread of the characteristic quantities consider by a wide range of single realisations. The inserted smoothing splines method has to satisfy two conditions.

- each homogeneous layer have to exist in the research area
- the homogeneous layer may not intersect

The sixth and seventh generated soil model based on this mentioned algorithm. The results at the modelled location are presented in table 5 columns 3 and 4. For the results in column 3 is used the reference profile from table 4 and for column 4 the reference profile from DS 1.

Table 6 shows a summary of the strata of the different homogeneous layers (L1-L3). Column 1-4 present the adaptive soil modelling. To prove the quality of the single simulation results on the foundation location column 5 shows the result of the measurement.

Table 5. Summary of the strata of the different soil models at the location [m]

	preliminary excavation	detail excavation	detail excavation consistent	detail excavation consistent	reality
	1	2	3	4	5
L 1	129,62	130,74	130,18	130,78	130,70
L 2	127,46	127,75	127,28	127,47	127,45
L 3	119,42	115,02	116,45	112,95	112,70

The geostatistical determination of the homogeneous profile, column 2, has a good validity in layer 1 and layer 2 to the reference profile, but the result is not consistent. The difference in the thickness from layer 1 to 3 can be expressed |15,72m – 18,00m| = 2,28m. This difference is not a measure for the quality of the geostatistical soil model.

The additional homogeneous profiles, like mentioned before, improved the estimation. The results get better with the increasing quality of the additional information, the inputted homogeneous profile. So the estimated stratum of layer 3 in column 3 is not better than in the steps before. Is it possible to integrate a reference profile in the near of the planed foundation location, reproduce the soil model a good agreement with the in-situ situation, column 4 and 5.

A geostatistical 3D-Model on the base of a given data set can be uses only for limited expressions for chosen single location. In the model important information to the trend and isotropy behaviour in the research area are integrated. These are not enough for expressions to small locations in the field. The additional integrated reference profile can improve the results regionally.

The results of this adaptive improvement used to design of an isolated machine footing at the reference point. The uncertainties and spread of integrated parameters will be involved by using a high number of realisations. As numerical model we use the Monte-Carlo Simulation (MCS).

4 DETERMINATION OF CHARACTERISTIC QUANTITIES

4.1 *General*

From the results of the cone penetration tests and the static sounding tests the resistance parameter q_{c1} is derived. The sounding results have been evaluated every 25cm and they have been summarized in one data set to one parent population.

Intermediate values are linear interpolated. As a result the in-situ soil is subdivided in three homogeneous layers, the homogeneous profile. For settlement analysis it is necessary to distinguish between the shear modulus for small strains and for normal strains, because the dependency between the resistance parameter q_{c1} and the shear modulus G_o exist only for small strains (figure 5 and figure 6). The Young's modulus E, the stiffness modulus E_s and the shear modulus G are linear dependent.

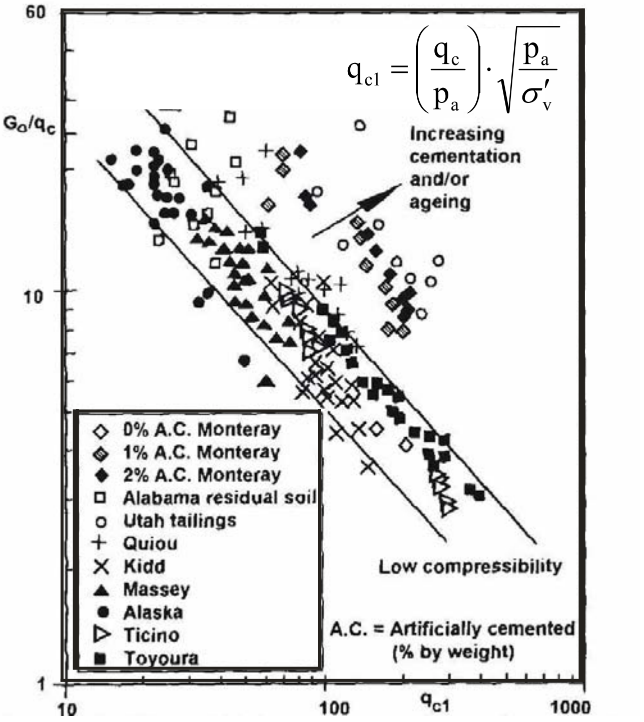


Figure 5. Ratio between tip pressures qc of CPT and the shear modulus G_o for small strains ROBERTSON 1997

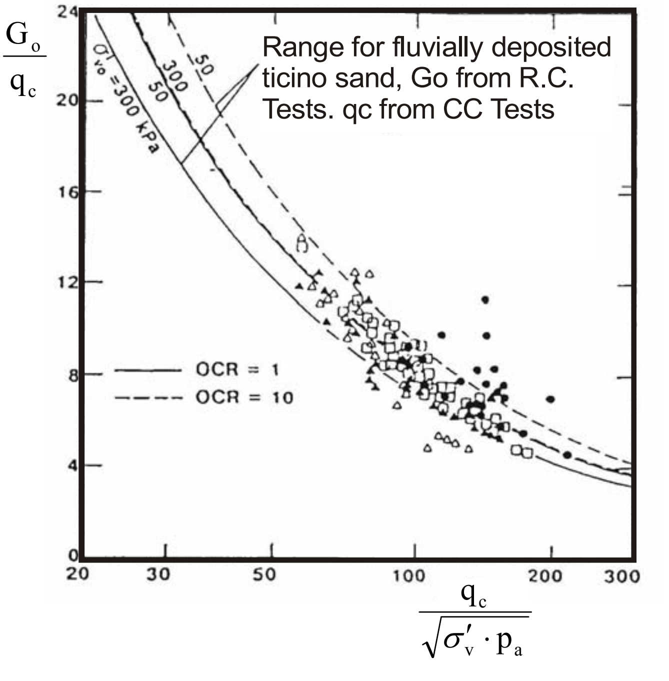


Figure 6. Ratio between tip pressures qc of CPT and the shear modulus G_o for small strains IMAI/TONOUCHU 1982

The shear modulus can be transformed into the young's modulus and in the stiffness modulus using a predefined Poisson's ratio ν. For all calculations we used an uniform ratio ν = 0.35.

$$G_o = \frac{E_o}{2\cdot(1+\nu)} \qquad (1)$$

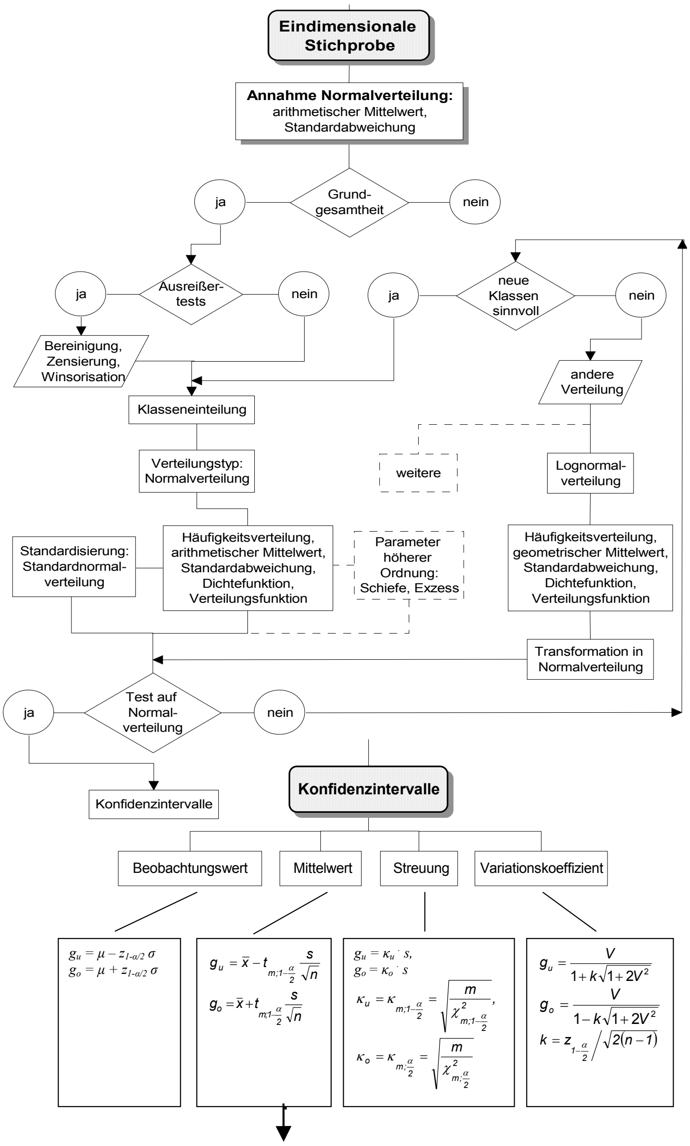
$$E_{so} = \frac{1-\nu}{1-\nu-2\cdot\nu^2}\cdot E_o = \frac{1-\nu}{1-\nu-2\cdot\nu^2}\cdot 2\cdot(1+\nu)\cdot G_o$$

$$E_{so} = \frac{2-2\nu^2}{1-\nu-2\cdot\nu^2}\cdot G_o$$

$$E_{so} = 4{,}33\cdot G_o \quad \text{oder} \quad \frac{1}{E_{so}} = 0{,}231\cdot\frac{1}{G_o} \qquad (2)$$

4.2 Characteristic Quantities

The data included in the different adaptive model steps are combined to a parent population. The characteristic quantities are determined by the scheme of Eurocode 7 (EC 7) and SCHUPPENER 1999. Under the assumption of normal distributed data the scheme is shown in figure 7.



$$\text{char } E_{so} = E\left[E_{so,i}\right] - t_{m,1-\frac{\alpha}{2}}\cdot\sqrt{\frac{E\left[X^2\right]}{m}} \qquad (3)$$

Figure 7. Determination of Characteristic Quantities

Without outliers and eliminated trends in the data set the characteristic quantities, the corresponding mean values and standard deviation are shown in table 6.

Table 6. Characteristic Quantities and statistical parameter for shear modulus G_o and the stiffness modulus E_s

	Layer 1	**Layer 2**	**Layer 3**
Shear Modulus G_o (NV) [MN/m^2] $E[x] / E[x]^2$	**25.017** 7.532	**32.300** 4.870	**115.540** 31.401
Shear Modulus G_o (LNV) [MN/m^2] $E[x] / E[x]^2$	**23.253** 5.317	**32.600** 4.840	**121.225** 34.456
Stiffness Modulus $E_{so} = 4{,}33 G_o$ (NV) $E[x] / E[x]^2$	**108.323** 32.613	**139.859** 21.087	**500.288** 135.966
Stiffness Modulus $E_{so}^{-1} = 0{,}23 G_o^{-1}$ (NV) $E[x] / E[x]^2$	**$1{,}02\cdot10^{-5}$** $2{,}92\cdot10^{-6}$	**$7{,}15\cdot10^{-6}$** $1{,}16\cdot10^{-6}$	**$2{,}00\cdot10^{-6}$** $6{,}54\cdot10^{-7}$
characteristic Quantity E_{so} [MN/m^2]	**98.407**	**136.063**	**486.590**

On the base of statistical tests the data sets are normal distributed. The density functions in the numerical simulation haven't a left side or a right side boundary. The randomly used parameter set for each calculation step is determined from the whole range. This assumption is possible, because the relation between the mean value E[X] and the accompanying standard deviation $E[x]^2$ is

$$E[x] - 3\cdot E[x]^2 \geq 0$$

The probability, that negative values will be simulated is less than 0,01%.

The following calculations of the skew position of the isolated foundation will use the characteristic quantities. Inside the Monte-Carlo simulation of the limit state function the random values will be determined using the mean value and the standard deviation of each involved data set.

5 FAILURE PROBABILITY OF FOUNDATION

5.1 *General*

For the design of the isolated machine foundation the bearing capacity and the deformation capability is to investigate. In this example we look for the ultimate serviceability limit state and define the skew position tanα as limit state.

5.2 *Limit state function*

The technical guidelines demand a small skew position. As limit state function we use the following settlement equation.

$$s_m + s_x = \sigma_o \cdot b \cdot \left[\frac{f_1}{E_{m,1}} + \sum_{i=2}^{n} \frac{\left(f_i - f_{i-1}\right)}{E_{m,i}} \right] + \frac{M_y}{b^3} \cdot \left[\frac{f_{x,1}}{E_{m,1}} + \sum_{i=2}^{n} \frac{\left(f_{x,i} - f_{x,i-1}\right)}{E_{m,i}} \right] \quad (4)$$

The constant immediate settlements s_m are not of interest. The limited state is defined to $\tan\alpha \leq 0{,}008$. The function can be simplify for the determination of the skew position to

$$s_x = \frac{b}{2} \cdot \tan\alpha = \frac{b}{2} \cdot \frac{M_y}{b^3} \cdot \left[\frac{f_{x,1}}{E_{m,1}} + \sum_{i=2}^{n} \frac{\left(f_{x,i} - f_{x,i-1}\right)}{E_{m,i}} \right] \quad (5)$$

or

$$g\left(\frac{1}{E_{mi}}\right) = \frac{M_y}{b^3} \cdot \left[\frac{f_{x,1}}{E_{m,1}} + \sum_{i=2}^{n} \frac{\left(f_{x,i} - f_{x,i-1}\right)}{E_{m,i}} \right] - 0{,}008 \quad (6)$$

For $g(1/E_{m,i}) \leq 0$ the construction fail and for $g(1/E_{m,i}) \geq 0$ the construction is reliable.

5.3 *Stiffness modulus*

The stiffness modulus and the shear modulus are elastic parameters and depend on the present strain state, YAMASHITA 2000. It is to decide between the stiffness modulus for small strains E_{so} and the stiffness modulus for normal strain E_s, SIMONS 2000.

With the relationship in figure 5 and figure 6 the stiffness modulus for small strains can be determined. As a function we use a bilinear relationship, figure 8. The reference stiffness modulus, determined in laboratory tests, is labelled as $E_{m,min}$.

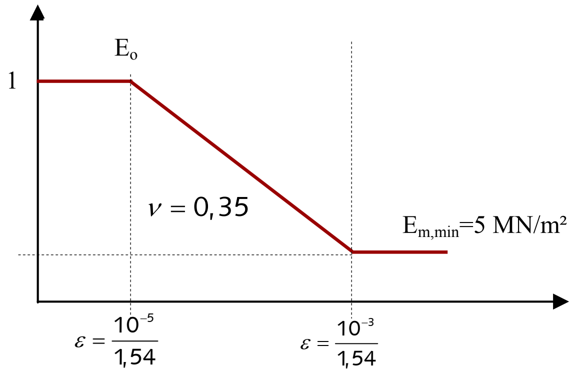


Figure 8. Empirical Relationship between the stiffness modulus and elastic strains SCHÖNHARDT 2002

The skew position of the foundation depends on the present stiffness modulus. The maximum of elastic strains is directly under the isolated foundation, so that the bilinear function reduced the stiffness modulus higher than in deeper layers. Therefore the homogeneous layer 1 and 2 are subdivided in 20 sections. For universal calculations we use a software application, developed on the professorship of foundation engineering. This program considers the characteristic quantities and the statistical parameter set of each data set.

5.4 *Failure probability*

The permitted failure probability p_f is subjective. Therefore we used the recommendation of Deutsches Institut für Normung 1981. On the base of safety classes the safety index β is classified.

Table 7. Safety index β and the corresponding failure probability for predefined safety classes

limit state	safety classes		
	1	2	3
deformation	2,5 $6{,}21 \cdot 10^{-3}$	**3,0** $\mathbf{1{,}35 \cdot 10^{-3}}$	3,5 $1{,}35 \cdot 10^{-4}$
bearing capacity	4,2 $1{,}34 \cdot 10^{-5}$	4,7 $1{,}30 \cdot 10^{-6}$	5,2 $9{,}98 \cdot 10^{-8}$

The failure probability is determined with the MCS-method, because the implemented variables are not correlated.

The value depends on the formulation of the limit state function g(x), the statistical parameter of the integrated variables, especially of the standard deviation $\sigma_{g(x)}$, and on the analytical or numerical calculation method SCHÖNHADT 2002.

$$P_f = \frac{1}{2 \cdot \pi} \cdot \int_{-\infty}^{0} e^{-\frac{1}{2} \cdot \left(\frac{g(X_i)}{\sigma_{g(X_i)}} - \beta \right)^2} d\left(\frac{g(X_i)}{\sigma_{g(X_i)}} \right) = \phi(-\beta) \quad (7)$$

For a given failure probability of $p_f = 1{,}35 \cdot 10^{-3}$ 10^5 realisations M are necessary. To improve the precision on the factor 10, 100 times more simulations are to do.

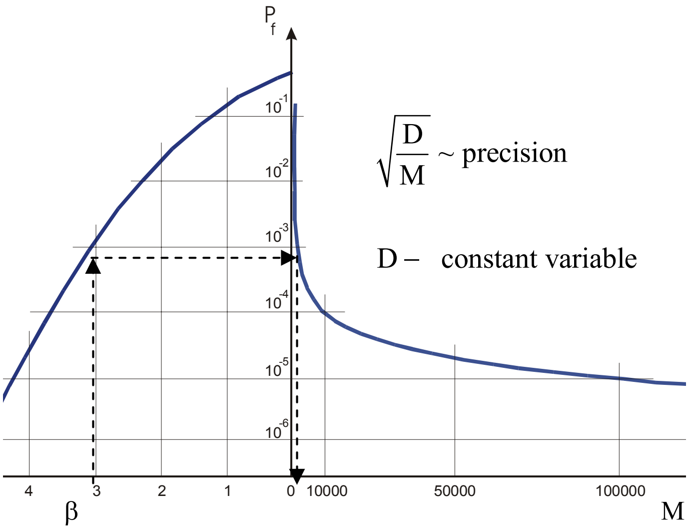


Figure 9. Dependency between failure probability p_f and the number of realisations M, MÜLLER 1979

5.5 *Example*

For example we consider a square sized spread machine footing loaded with

- one axial moment m = 500 kNm
- centrically vertical load n =1.200 kN

The surface has a stratum of 127,50m. We have assumed that no ground improvement will be done. In table 8 the necessary foundation dimension for a quadratic geometry are summarized.

Table 8: Dimensions b of a quadratic shallow foundation in dependency of the adaptive ground modelling and the deviation of the single results to the reference location in Table 6.

	preliminary investigation	detail investigation	detail investigation consistent	detail investigation consistent	reality
	1	2	3	4	5
b	2,630	2,605	3,205m	3,530m	3,450m
Δ	23,8%	24,5%	7,2%	2,3%	0,0 %

These foundation dimensions are determined by 10^6 realisations using the different prediction of soil modelling. In this example the incomplete knowledge of the real strata and stiffness of the layers lead to an underestimation of the necessary dimensions to guarantee the allowable relative settlements. The error is expressed in terms of linear difference of the footing size.

6 CONCLUSION

Because of the genesis the description and modelling of the in-situ soil is complicate. Therefore the characteristic quantities of the soil are containing uncertainties and spatial variations. Due limited budgets there are often only few data from soil investigation. Ground models, based on small data sets, will produce inadequate estimations.

With the use of geostatistical methods we can produce better estimations between the given data set than with interpolation. A homogeneous profile at certain location can be determined.

An important aspect for using geostatistical models is the determination of anisotropy and the trend behaviour. Without this knowledge all estimations could not be better than a subjective estimation of expert.

The presented 3d-homogeneous model contains information to correlations between the integrated quantities, the regionally distribution, the anisotropy and the trend behaviour. Nevertheless this model is inconsistent to the reference results. Therefore the method of smoothing splines is additional integrated. The comparison of model results with the realistic profile is able to improve the estimation of the homogeneous model. Alternative there exist other polynomial methods. The use of them depends on the actual boundary conditions.

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